CORE MATHEMATICS (C) UNIT 1 TEST PAPER 7

1. If $x^p = (\sqrt{x})^3$, $x^q = \dfrac{1}{x^2}$ and $x^r = \dfrac{x^p}{x^q}$,

   (i) find the value of r, [3]

   (ii) evaluate x^r when $x = 4$. [2]

2. Given that $7 - 4x - x^2 \equiv m - (x + n)^2$,

   (i) find the values of the constants m and n. [4]

   (ii) Deduce the largest value of k for which the equation $7 - 4x - x^2 = k$ has at least one real root. [2]

3. A container, initially empty, is filled with liquid. At time t seconds after filling starts, the volume of liquid is V cm^3 where $V = \dfrac{\pi t}{2}(t + 3)$.

   (i) Find, as a multiple of π, the rate at which V is increasing at the instant when $t = 4$. [4]

   (ii) Find the second derivative of V with respect to t. [2]

4. The line l has equation $y = x + 2$ and the circle C has equation $x^2 + y^2 + 3x - 4y = 1$.

(i) Find the coordinates of the points where l and C intersect. [6]

(ii) Determine, by calculation, whether or not l passes through the centre of C. [2]

5. The points P and Q have coordinates (4, 5) and (–2, –7) respectively.

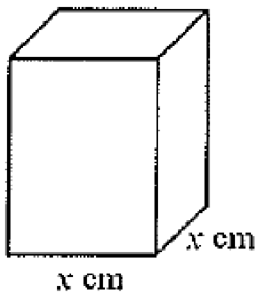
(i) Find the length of the line segment PQ, giving your answer in the form $k\sqrt{p}$ where p is prime. [3]

(ii) Find equations for the following straight lines:

(a) the line through the origin which is parallel to PQ, [2]

(b) the line through the mid-point of PQ which is perpendicular to PQ. [4]

6. A piece of thin wood, of length 240 cm, is used to make the twelve edges of a rectangular box with a square base of side x cm.



(i) Show that the volume of the box is $(60x^2 - 2x^3)$ cm^3. [4]

(ii) Find the value of x for which the volume is maximum. [5]

(iii) Describe the type of box for which the volume is maximum. [1]

7. (i) Find the three values of x for which $x^3 - x = 0$. [4]

(ii) Find the coordinates of the stationary points of the function $x^3 - x$. [4]

(iii) Sketch the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = x^3 - x$. [2]

(iv) On separate diagrams, sketch the curves with equations

(a) $y = \mathrm{f}(x - 1)$, (b) $y = -\mathrm{f}(x)$,

showing in each case the stationary points and the coordinates of any points of intersection with the axes. [4]

8. The line l has equation $y = 6 - x$. The curve C has equation $y = x^2 - 4x + 2$.

(i) Find the coordinates of the point on C at which the tangent is parallel to l. [4]

(ii) Find the coordinates of the points where l and C intersect. [4]

(iii) Write down the solution set of the inequality $x^2 - 4x + 2 < 6 - x$. [3]

(iv) Find the x-coordinates of the points where C crosses the x-axis, giving your answers in surd form simplified as far as possible. [4]

CORE MATHS 1 (C) TEST PAPER 7 : ANSWERS AND MARK SCHEME

1.	(i) $p = 3/2,\ q = -2$ $\quad r = p - q = 7/2$	B1 B1 B1	
	(ii) $4^{7/2} = 2^7 = 128$	M1 A1	5
2.	(i) $-(x^2 + 4x - 7) = -((x+2)^2 - 11) = 11 - (x+2)^2$ $\quad m = 11, n = 2$	M1 M1 A1 A1	
	(ii) Equation is $(x+2)^2 = 11 - k$, so largest k is 11	M1 A1	6
3.	(i) $V = \frac{\pi t^2}{2} + \frac{3\pi t}{2}$ $\quad \frac{dV}{dt} = \pi t + \frac{3\pi}{2} = \pi\left(4 + \frac{3}{2}\right) = \frac{11\pi}{2}$ when $t = 4$	B1 M1 A1 A1	
	(ii) Second derivative of $V = \pi$	M1 A1	6
4.	(i) $x^2 + (x+2)^2 + 3x - 4(x+2) - 1 = 0$ $\quad 2x^2 + 3x - 5 = 0$	M1 A1 A1	
	$(2x+5)(x-1) = 0$ $\quad (1, 3),\ (-5/2, -1/2)$	M1 A1 A1	
	(ii) Centre is $(-3/2, 2)$, which does not lie on l	M1 A1	8
5.	(i) $PQ^2 = 36 + 144 = 180$ $\quad PQ = 6\sqrt{5}$	M1 A1 A1	
	(ii) (a) Gradient $PQ = 2$, so line is $y = 2x$	M1 A1	
	(b) Mid-point is $(1, -1)$ $\quad$ Grad $= -1/2$ $\quad y + 1 = -1/2\,(x - 1)$	B1 B1 M1 A1	9
6.	(i) $8x + 4h = 240$ so $h = 60 - 2x$ $\quad V = x^2(60 - 2x) = 60x^2 - 2x^3$	M1 A1 M1 A1	

6.	(i)	$8x + 4h = 240$ so $h = 60 - 2x$ $\quad V = x^2(60 - 2x) = 60x^2 - 2x^3$	M1 A1 M1 A1	
	(ii)	$dV/dx = 120x - 6x^2 = 0$ when $x = 20$	M1 A1 M1 A1	
	(iii)	Thus the volume is greatest when the box is a cube	A1	9
7.	(i)	$x(x-1)(x+1) = 0$ $\quad x = -1, x = 0, x = 1$	M1 A1 A1 A1	
	(ii)	$3x^2 - 1 = 0$ $\quad x = \pm 1/\sqrt{3}$ $\quad y = \mp 2/(3\sqrt{3})$	M1 A1 M1 A1	
	(iii)	Cubic curve through (–1, 0), (0, 0), (1, 0)	B2	
	(iv)	(a) Cubic curve through (0, 0), (1, 0), (2, 0) (b) Reflected in x-axis	B2 B2	14
8.	(i)	$2x - 4 = -1$ so $x = 3/2$ $\quad$ Point is $(3/2, -7/4)$	M1 A1 M1 A1	
	(ii)	$x^2 - 3x - 4 = 0$ $\quad (x+1)(x-4) = 0$ $\quad (-1, 7), (4, 2)$	B1 M1 A1 A1	
	(iii)	Curve lies below line when $-1 < x < 4$	M1 A1 A1	
	(iv)	$(x-2)^2 - 2 = 0$ $\quad x = 2 - \sqrt{2}, x = 2 + \sqrt{2}$	M1 A1 A1 A1	15